# Exercise 86

For the following exercises, for each polynomial, a. find the degree; b. find the zeros, if any; c. find the $y$-intercept(s), if any; d. use the leading coefficient to determine the graph's end behavior; and e. determine algebraically whether the polynomial is even, odd, or neither.

$$f(x) = x^3 + 3x^2 - x - 3$$

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**Solution**

## Part (a)

The degree of the polynomial is 3 because the highest power of $x$ is 3.

## Part (b)

Set $f(x) = 0$.

$$f(x) = x^3 + 3x^2 - x - 3 = 0$$

Observe that if $x = 1$, then the function evaluates to zero:

$$f(1) = (1)^3 + 3(1)^2 - 1 - 3 = 0.$$

This means that $x - 1$ is a factor of $f(x)$.

$$\begin{aligned} f(x) &= (x-1)\left(\frac{x^3 + 3x^2 - x - 3}{x-1}\right) \\ &= (x-1)(x^2 + 4x + 3) \\ &= (x-1)(x+1)(x+3) \end{aligned}$$

Therefore, the zeros are

$$x = \{-3, -1, 1\}.$$

## Part (c)

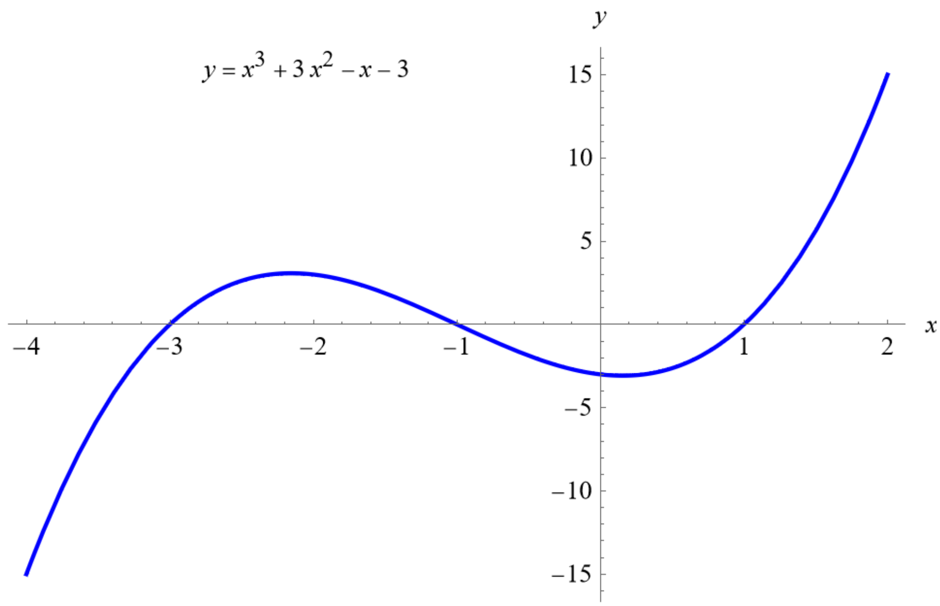
$y$-intercepts are the points where the function touches the $y$-axis, which occurs when $x = 0$.

$$f(0) = (0)^3 + 3(0)^2 - (0) - 3 = -3$$

Therefore, there's one $y$-intercept: $(0, -3)$.

## Part (d)

$x^3$ is the dominant term in the polynomial, so the graph is cubic. Since the coefficient is +1, it goes down to the left and goes up to the right. The graph of $f(x)$ versus $x$ below illustrates this.




## Part (e)

Plug in $-x$ for $x$ in the function.

$$\begin{aligned} f(-x) &= (-x)^3 + 3(-x)^2 - (-x) - 3 \\ &= -x^3 + 3x^2 + x - 3 \end{aligned}$$

Since $f(-x) \neq f(x)$, the function $f(x)$ is not even.

Since $f(-x) \neq -f(x)$, the function $f(x)$ is not odd.